On Efficient Management of Complex Systems

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Complex systems profoundly change human activities of the day. However, it is still unknown how to deal with complex systems efficiently without confronting NP-hard problems [1]. Existing concepts of complexity are very important on many occasions, but they do not explain how the performance of a system is dependent on its complexity and whether efficient management of complex systems may be possible at all.

To address the situation we consider the description of complex systems in terms of self-organization processes of prime integer relations [2]. The description is realized through the unity of its two equivalent forms, i.e., arithmetical and geometrical. In the arithmetical form a complex system is characterized by nonlocal correlation structures defined through self-organization processes of prime integer relations. In the geometrical form the self-organization processes of prime integer relations become isomorphically represented by certain transformations of two-dimensional patterns determining the dynamics of the complex system.

In preserving a complex system as a whole the equivalence of the forms unites the dynamics and the structure of the system. The dynamics of a complex system is determined by the spacetime patterns of the parts to fit precisely into the geometrical patterns of the system. If this condition is not met, then one or more of the relationships provided by the prime integer relations for the correlation structures disappear and the complex system collapses.

The self-organization processes of prime integer relations start with different integers and progress to different levels as a result producing a hierarchical complexity order. A concept of complexity, called the structural complexity, is introduced to measure the complexity of a system in terms of self-organization processes of prime integer relations [2]. In particular, the higher the level self-organization processes of prime integer relations progress to, the greater is the structural complexity of the system, i.e., under the processes at each level the parts combine and make up more complex parts at the higher level.

By computational experiments we investigate whether the performance of an optimisation algorithm for a NP-hard problem might behave as a concave function of the algorithm's structural complexity [3]. In particular, we propose an optimisation algorithm $\mathcal{A}$, as a complex system, of N computational agents minimising the average distance in dealing with the traveling salesman problem. The extensive experiments are conducted by using a set $\mathcal{P}$ of the benchmark traveling salesman problems [4]. In the optimisation algorithm $\mathcal{A}$ all agents start in the same city and each agent at each step visits the next city by using one of the two strategies: a random strategy, i.e., visit the next city at random, or the greedy strategy, i.e, visit the next closest city.

The state of the agents solving a problem with n cities at step $j = 1, ..., n-1$ can be described by a binary sequence $s_j = s_{1j}...s_{Nj}$, where $s_{ij} = +1$, if agent $i = 1, ..., N$ uses the random strategy and $s_{ij} = -1$, if the agent i uses the greedy strategy. The dynamics of the complex system is realized as step by step the agents choose their strategies and can be encoded by an $N \times (n-1)$ binary strategy matrix

$$S = \{s_{ij}, i = 1, ..., N, j = 1, ..., n-1\}.$$

We aim to monotonically change the structural complexity of the optimisation algorithm $\mathcal{A}$ forcing it to make the transition from regular behavior to chaos by period-doubling. In attempt to realize that we introduce a control parameter $v, 0 \leq v \leq 1$ and change it from 0 to 1 so each agent could mimic the period-doubling route to chaos. For this purpose in choosing the next strategy each agent follows an optimal if-then rule. The rule relies on the Prouhet-Thue-Morse (PTM) sequence

$$+1 - 1 - 1 + 1 - 1 + 1 + 1 - 1 \ ...$$

and has the following description:

1. if the last strategy is successful, continue with the same strategy.

2. if the last strategy is unsuccessful, consult PTM generator which strategy to use next.

The change of the control parameter v from 0 to 1 defines a flow in the space of the strategy matrices. The flow is controlled to have the property of monotonicity of the structural complexity. Following such a flow the correlation structures of a system experience certain phase transitions, i.e., the parts of the system combine to make up more complex parts and so on.

Remarkably, for each problem $p \in \mathcal{P}$ tested we find that the performance of the optimisation algorithm $\mathcal{A}$ behaves as a concave function of the control parameter v with the only global maximum at a value $v^*(p)$. We use the global maximums $\{v^*(p), p \in \mathcal{P}\}$ to probe whether an optimality condition determining that for the optimal performance the structural complexities of the algorithm $\mathcal{A}$ and the problem have to be properly related exists. In particular, from the performance global maximums $\{v^*(p), p \in \mathcal{P}\}$ we get the strategy matrices $\{S(v^*(p)), p \in \mathcal{P}\}$ and approximate the structural complexities of the optimisation algorithm $\mathcal{A}$ and a problem p as follows.

The structural complexity $\mathbf{C}(A(p))$ of the algorithm $\mathcal{A}$ is approximated by the quadratic trace

$$C(\mathcal{A}(p)) = \frac{1}{N^2} tr(V^2(v^*(p))) = \frac{1}{N^2} \sum_{i=1}^{N} \lambda_i^2$$

of the variance-covariance matrix $V(v^*(p))$ obtained from the strategy matrix $S(v^*(p))$, where $\lambda_i, i = 1, ..., N$ are the eigenvalues of $V(v^*(p))$. The structural complexity $\mathbf{C}(p)$ of the problem p is approximated by the quadratic trace

$$C(p) = \frac{1}{n^2} tr(M^2(p)) = \frac{1}{n^2} \sum_{i=1}^{n} (\lambda'_i)^2$$

of the normalized distance matrix

$$M(p) = \{d_{ij}/d_{max}, i, j = 1, ..., n\},$$

where $\lambda'_i, i = 1, ..., n$ are the eigenvalues of $M(p)$, d_{ij} is the distance between cities i and j and d_{max} is the maximum of the distances.

To reveal the optimality condition we consider the points with the coordinates

$$\{x = C(p), y = C(\mathcal{A}(p)), p \in \mathcal{P}\}.$$

The result of their analysis points to a linear relationship between the structural complexities and suggests an optimality condition of the algorithm $\mathcal{A}$:

If the algorithm $\mathcal{A}$ demonstrates the optimal performance for a problem p, then the structural complexity $C(\mathcal{A}(p))$ of the algorithm $\mathcal{A}$ is in the linear relationship

$$C(\mathcal{A}(p)) = 0.67C(p) + 0.33. \qquad (1)$$

with the structural complexity $C(p)$ of the problem p.

Remarkably, the optimality condition combines the dynamics and structure. Namely, if the optimal performance is in place, then according to (1) the structural complexity equates the dynamics of the algorithm $\mathcal{A}$ to the structure of the problem p, i.e., the distance network with the vertices as the cities and the edges specifying the pairwise distances.

The computational experiments raise the possibility of a general optimality condition of complex systems:

If the structural complexity of a system is in a certain relationship with the structural complexity of a problem, then the complex system shows the optimal performance for the problem.

The general optimality condition presents the structural complexity of a system as a key to its optimisation. From its perspective the optimisation of a system could be all about the control of the structural complexity of the system to make it consistent with the structural complexity of the problem. It would help to know the structural complexity of the problem before the actual computations. In this case the optimality condition could specify the structural complexity of the system, which determines its optimal performance. Then, to obtain the optimal result, the control could be simply tuned for the system to operate with the required structural complexity.

Importantly, the experiments indicate that the performance of a complex system may indeed behave as a concave function of the structural complexity. Therefore, once the structural complexity could be controlled as a single entity, the optimisation of a complex system would be potentially reduced to a one-dimensional concave optimisation irrespective of the number of variables involved its description. This might open a way to efficient management of complex systems.

The results also point that classical optimisation algorithms may well approximate efficient quantum algorithms and thus suggest a new perspective on the connection between the classical and quantum domains. In particular, we note that the optimisation algorithm $\mathcal{A}$ shares a common feature with Shor's algorithm, which also significantly relies on the PTM sequence [5].

[1] J. Lanier, *The Complexity Ceiling*, in The Next Fifty Years: Science in the First Half of the Twenty-First Century, J. Brockman (Ed.), Phoenix, 2003, pp. 217-229.

[2] V. Korotkikh, *A Mathematical Structure for Emergent Computation*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1999; V. Korotkikh and G. Korotkikh, *Description of Complex Systems in terms of Self-Organization Processes of Prime Integer Relations*, in Complexus Mundi: Emergent Patterns in Nature, M. M. Novak (Ed.), World Scientific, New Jersey/London, 2006, pp. 63-72, arXiv:nlin.AO/0509008; V. Korotkikh and G. Korotkikh, *On an Irreducible Theory of Complex Systems*, InterJournal of Complex Systems, 2006, 1751, arXiv:nlin.AO/0606023.

[3] V. Korotkikh, G. Korotkikh and D. Bond, *On Optimality Condition of Complex Systems: Computational Evidence*, arXiv:cs.CC/0504092.

[4] G. Reinelt, *TSPLIB*, version 1.2 (accessed 28/11/2000).

[5] K. Maity and A. Lakshminarayan, *Quantum Chaos in the Spectrum of Operators Used in Shor's Algorithm*, arXiv:quant-ph/0604111.