A Genetic Algorithm for designing low-cost complex networks resilient to targeted attacks

George Leu
Akira Namatame

National Defense Academy of Japan

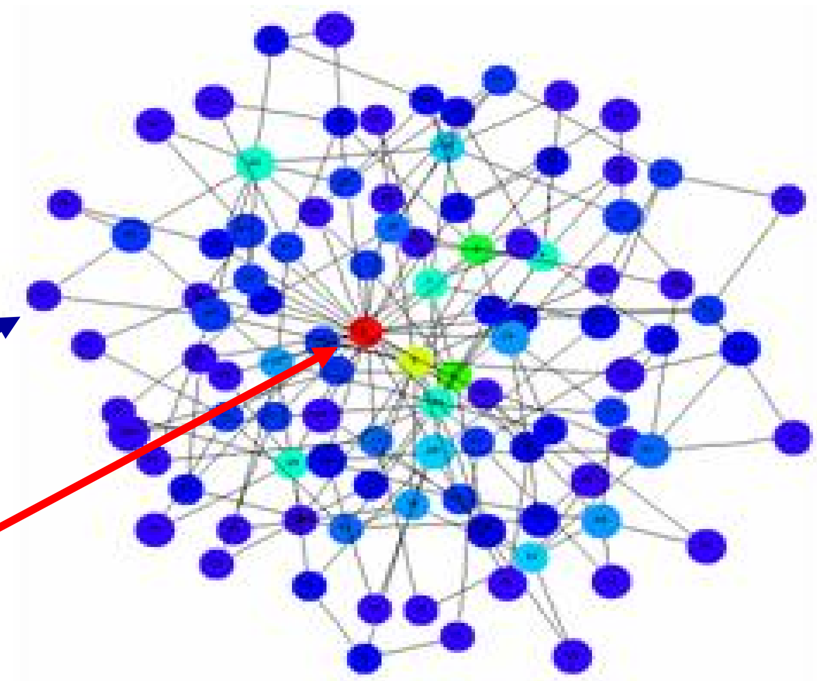
ARC Centre for Complex Systems
School of ITEE | The University of Queensland | ST LUCIA QLD 4069 | AUSTRALIA

T: +61 7 3365 1003 | F: +61 7 3365 1533 | E: outreach@accs.edu.au

intro

- Purpose:
  - design low-cost scale-free networks resilient to targeted attacks
  - optimize networks' robustness with minimal changes of the network's structure (minimize the optimization costs)
- Contents:
  - Complex networks & multi-objective optimization
  - GA
  - Results
  - conclusions

Complex networks:



(internet, transportation networks … etc)

- **power low** distribution of node degrees

    - most of the nodes = low degree

    - a few nodes = high degree (HUB nodes)

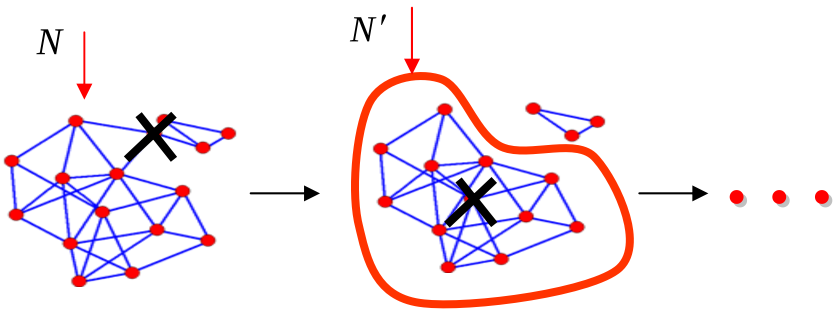
Scale-Free networks:

Vulnerable to HUB nodes failures/attacks

- very fast disintegration of the whole network

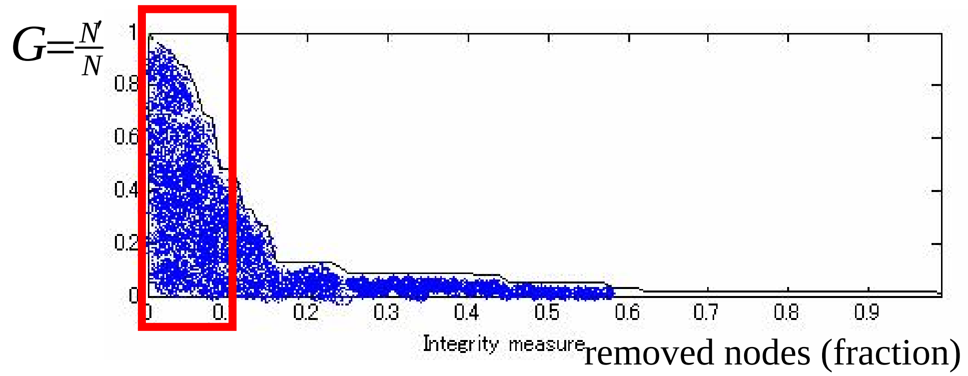
Improve SCALE-FREE networks' resilience to attacks on HUB nodes

1st objective: - improve resilience to targeted attacks



Targeted attack

Remove nodes in decreasing order of their connectivity



$G = \frac{N'}{N}$ = ***topological integrity***; G<1

N' = number of nodes in the largest connected network, after the attack

N = initial number of nodes

- remove first 10% most connected nodes
- maximize aria:

$$f_{obj1} = \sum_{upto10\%} G = \sum_{upto10\%} \frac{N'}{N}$$

The most resistant network: - an almost full connected graph – IMPOSSIBLE in real world.

2nd objective: - minimize the implementation cost

- E = number of edges in the graph
- consider that ***minimize cost ~ minimize number of edges***

$$f_{obj2} = \frac{E}{E_{max}}$$

The cheapest network: - a tree structure – resilience to targeted attacks is very low.

1st & 2nd objective: - improve network's resistance BUT keep the implementation costs in reasonable limits

$$f_{obj1} = \sum_{upto10\%} \frac{N'}{N} \quad \textbf{max}$$

$$f_{obj2} = \frac{E}{E_{\max}} \quad \textbf{min}$$

$$? \; f_{obj} = w * \frac{1}{f_{obj1}} + (1-w) * s * f_{obj2}$$

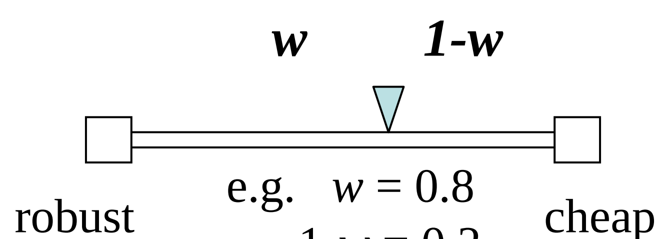
s - scaling coefficient.

- brings the 2 functions in the same variation range

$$\frac{1}{f_{obj1}} \cong s * f_{obj2}$$

w - 'weighting coefficient'.

- decide the importance of each objective



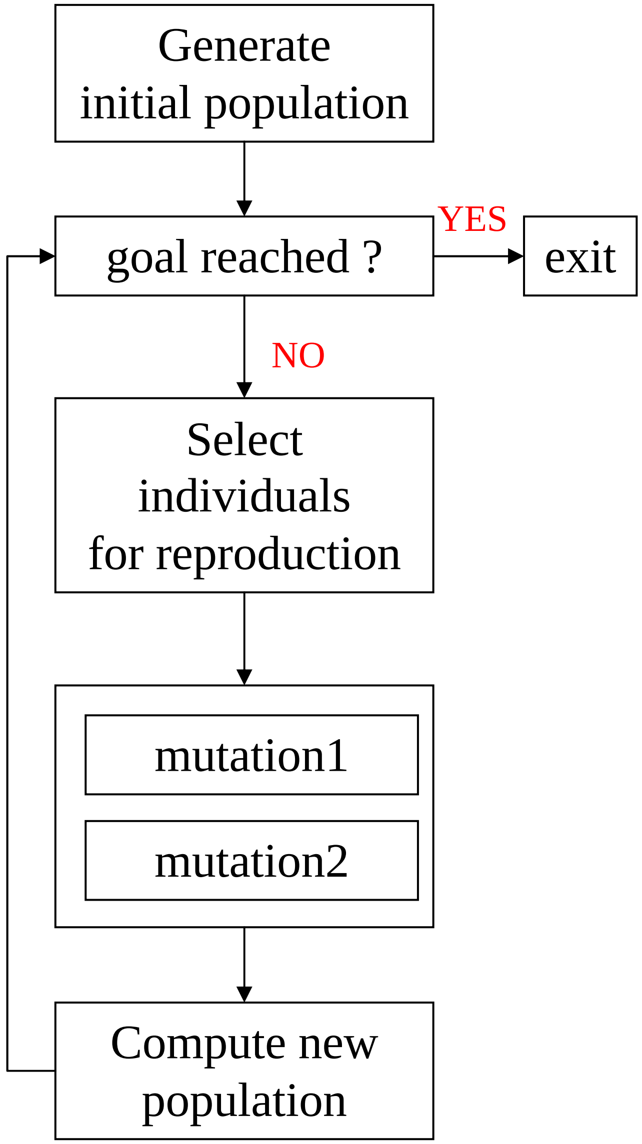
e.g. $w = 0.8$

$1\text{-}w = 0.2$

The overall objective function:

minimize
$$f_{obj} = w \cdot \frac{1}{\sum_i \frac{N_i'}{N}} + (1-w) \cdot s \cdot \frac{E}{E_{\max}}$$

G.A. structure:



- each individual in the population is a network
- the algorithm stops after a fixed number of generations
- random individuals
- generate new individuals
- **reinsert** new individuals according to their performances
- performances are calculated regarding the objective function

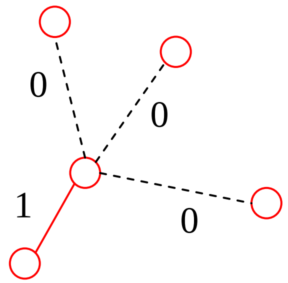
Input data:

- number of networks/population: **10 networks/pop**
- the networks have **power-low distribution of degrees** (generated using Preferential Attachment)
- number of nodes for each network: **500 NODES**
- number of generations: **50 generations**

Encoding:

- the networks are represented using Adjacency Matrix
- the genetic operators are applied inside of the A.M., directly

$$A_{ij} = \begin{cases} 1 & \text{link } ij \text{ exists} \\ 0 & \text{no link } ij \end{cases}$$



Genetic operators:

Mutation1:

- choose one node 'i' - randomly
- choose one of its stubs - randomly
- choose to modify the stub, or not - randomly

- provides
  - smooth movement in the space of solutions
  - fine adjustments for finding a local minimum

Mutation2:

- choose one node 'i' - randomly
- take all its possible stubs, one by one
  - choose to modify the stub, or not - randomly

- provides
  - large jumps in the space of solutions
  - a wide search for the global minimum

Reinsertion & objective function:

- compute new population using reinsertion based on ROULETTE METHOD

- objective function:

$$f_{obj} = w \cdot \frac{1}{f_{obj1}} + (1-w) \cdot s \cdot f_{obj2}$$

Results

<u>1st objective only:</u>

$w = 1$

$(1-w) = 0$

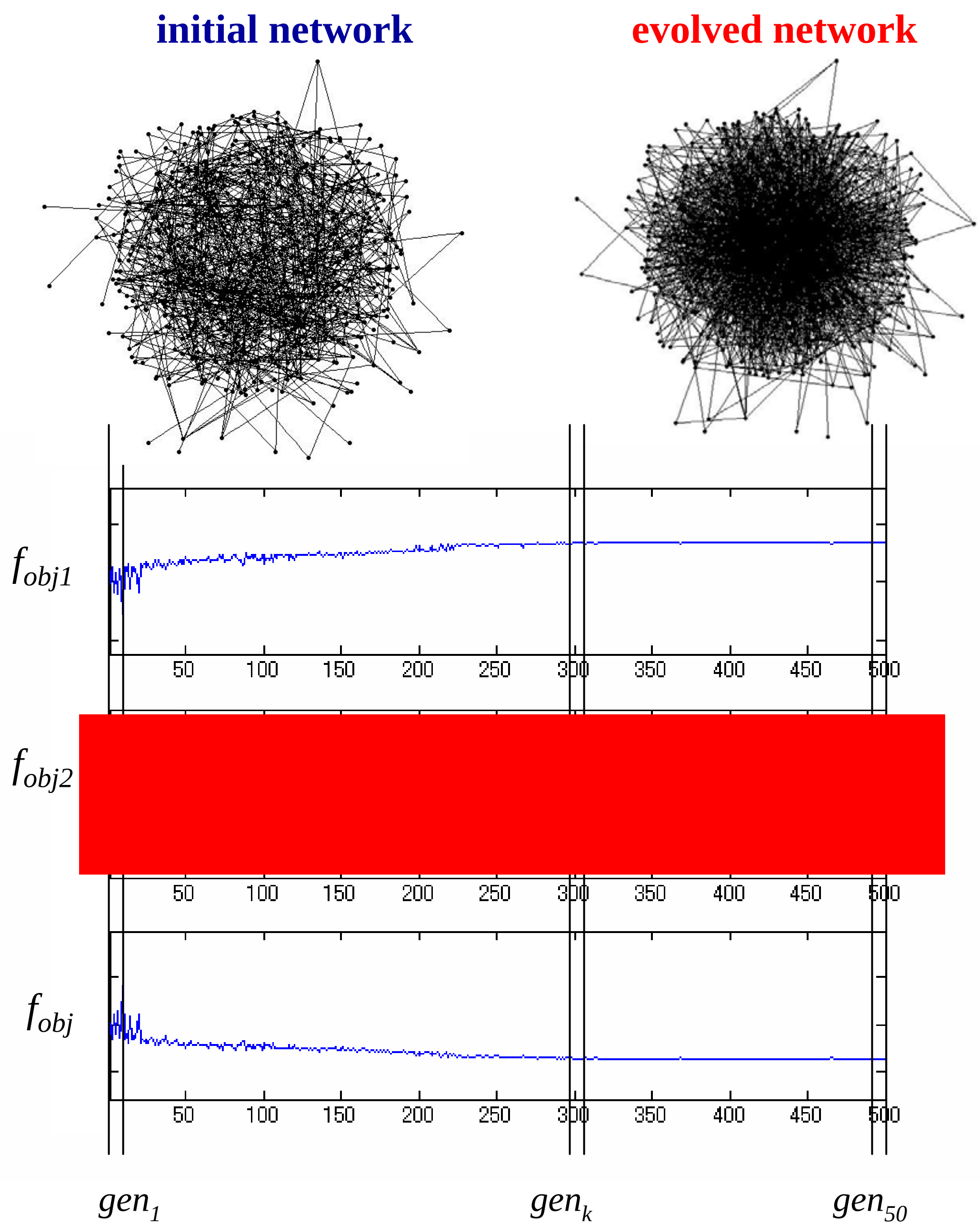
Review the objective function !!

$$f_{obj} = w \cdot \frac{1}{\sum_i \frac{N_i'}{N}} + (1-w) \cdot s \cdot \frac{E}{E_{\max}}$$

$$f_{obj} = w \cdot \frac{1}{f_{obj1}} + (1-w) \cdot s \cdot f_{obj2}$$

the population of networks evolves as follows:

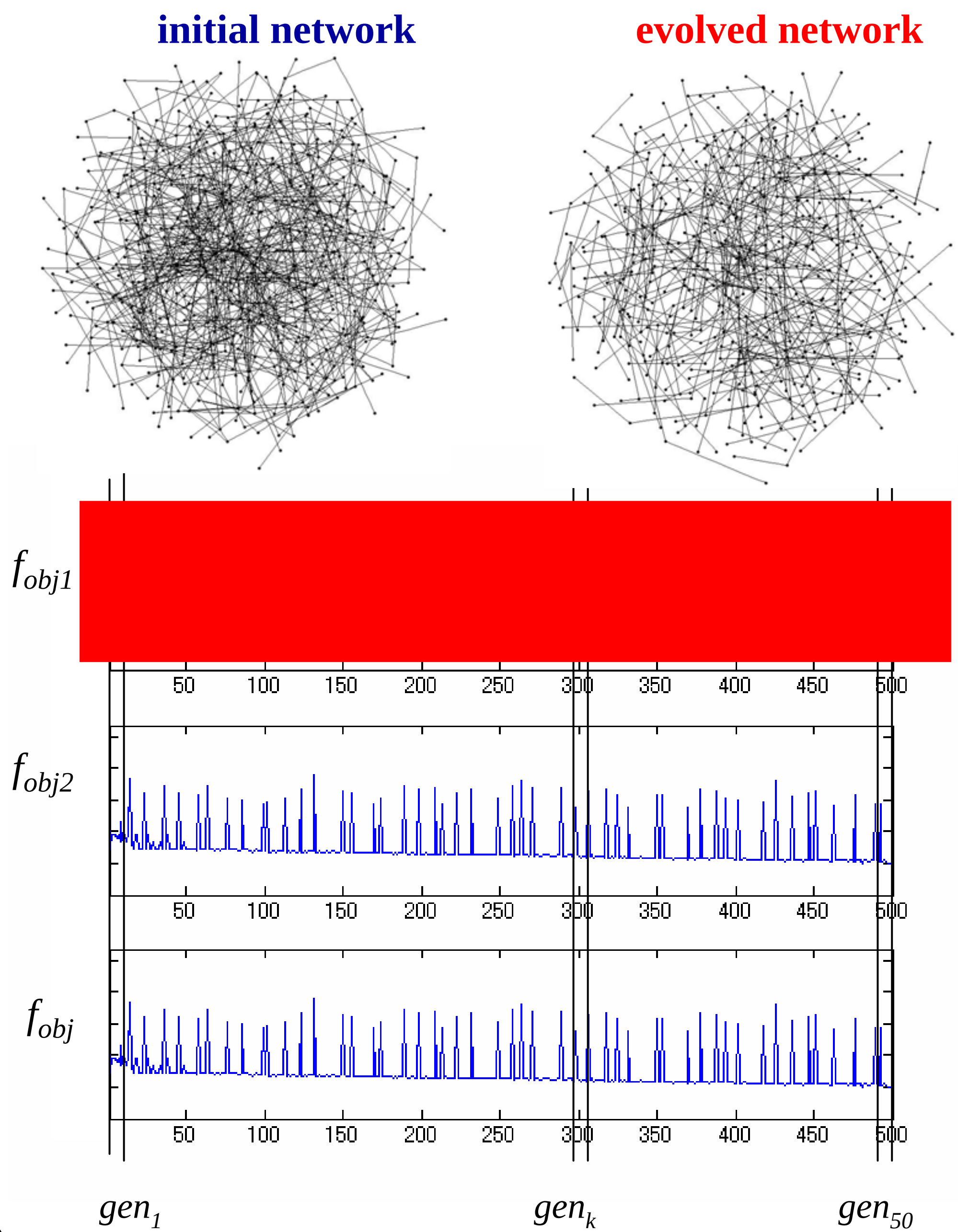
- first objective (network robustness):
    - increases visible
- second objective (number of edges):
    - dramatically INCREASES
- overall obj-function:
    - first objective's opposite variation



Performance of each network:

- from 1st to 50th generation
- 500 networks, in total

Results



Performance of each network:

- from 1st to 50th generation
- 500 networks, in total

<u>**2nd objective only:**</u>

***w* = 0**

***(1-w)* = 1**

Review the objective function !!

$$f_{obj} = w \cdot \cancel{\frac{1}{\sum_i \frac{N_i'}{N}}} + (1-w) \cdot s \cdot \frac{E}{E_{\max}}$$

$$f_{obj} = w \cdot \cancel{\frac{1}{f_{obj1}}} + (1-w) \cdot s \cdot f_{obj2}$$

the population of networks evolves as follows:

- first objective:
    - overall decreasing, but anyway chaotic-like
- second objective:
    - decreases to the lower limit (tree-like structure)
- overall obj-function:
    - identical with the second objective

Results

<u>**1^{st} & 2^{nd} objective**</u>

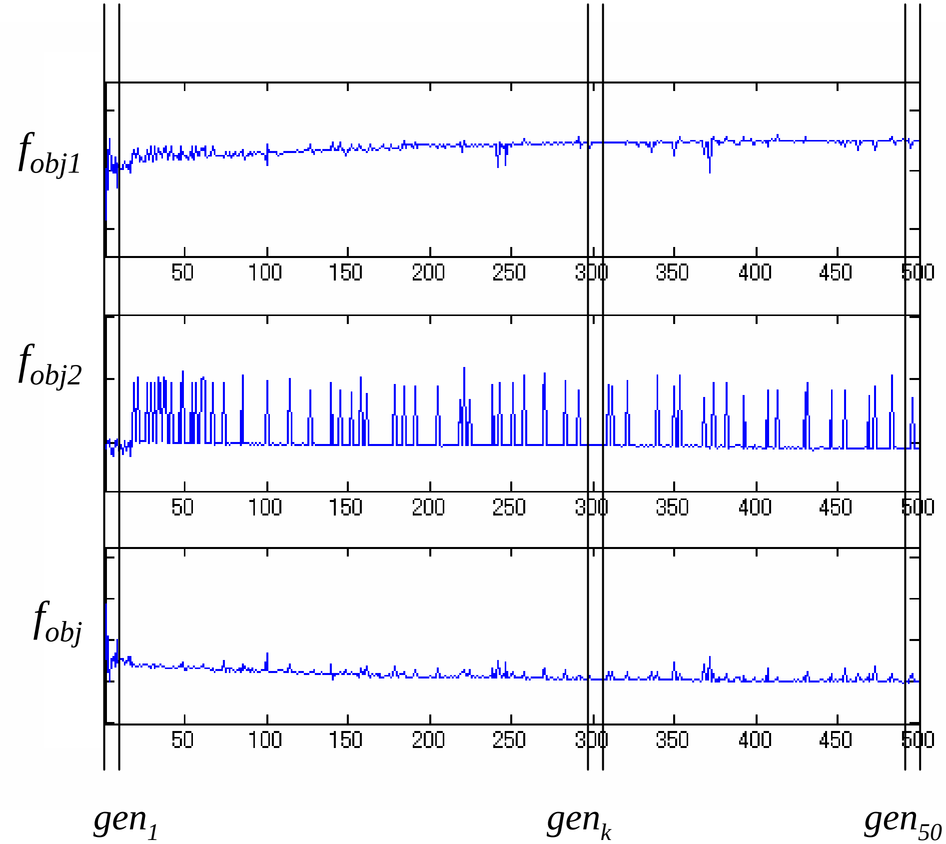
w = 0.5

$(1-w)$ = 0.5

Review the objective function !!

$$f_{obj} = w1 \bullet \frac{1}{\sum_i \frac{N_i'}{N}} + w2 \bullet s \bullet \frac{E}{E_{max}}$$

$$f_{obj} = w_1 \bullet \frac{1}{f_{obj1}} + w_2 \bullet (s \bullet f_{obj2})$$



Performance of each network:

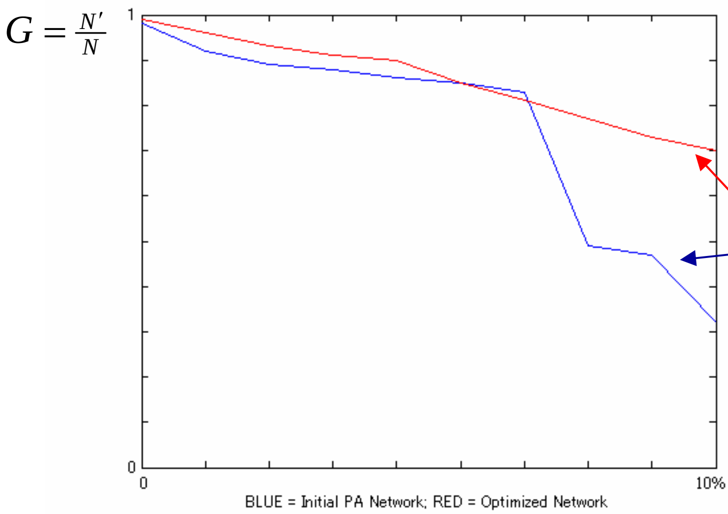
- from 1^{st} to 50^{th} generation
- 500 networks, in total

the population of networks evolves as follows:

- first objective:
  - increases visible
- second objective:
  - small increase, but almost constant overall
- overall obj-function:
  - decreases visible

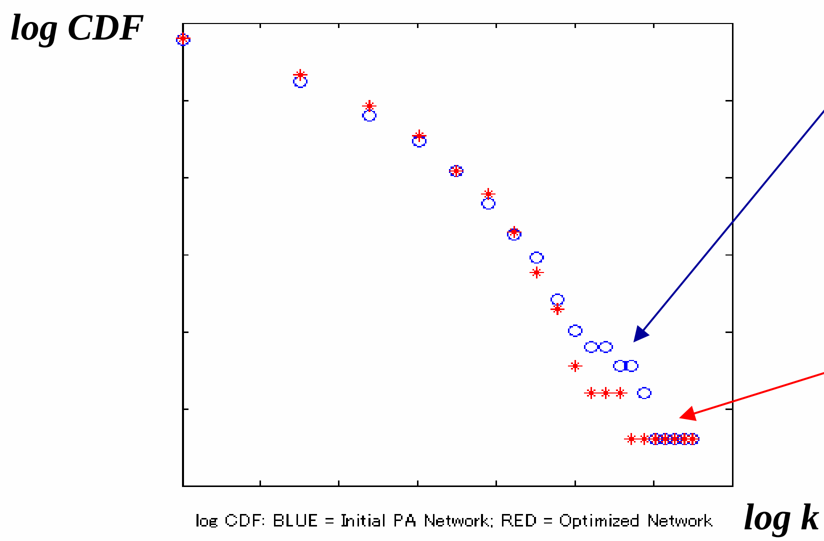
Should we expect for the network population to evolve into networks with the same structure, but better resilience ??

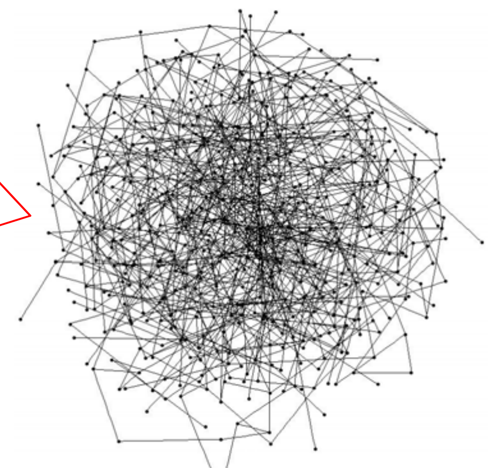
- **Sample network from the last generation has:**
    - higher resilience to targeted attacks
    - almost the same degree distribution
    - developed networks are of the same type with the old ones



resilience to targeted attacks

BLUE - initial network





RED - evolved network

Results

<u>1st & 2nd objective</u>

Best result as far as now:

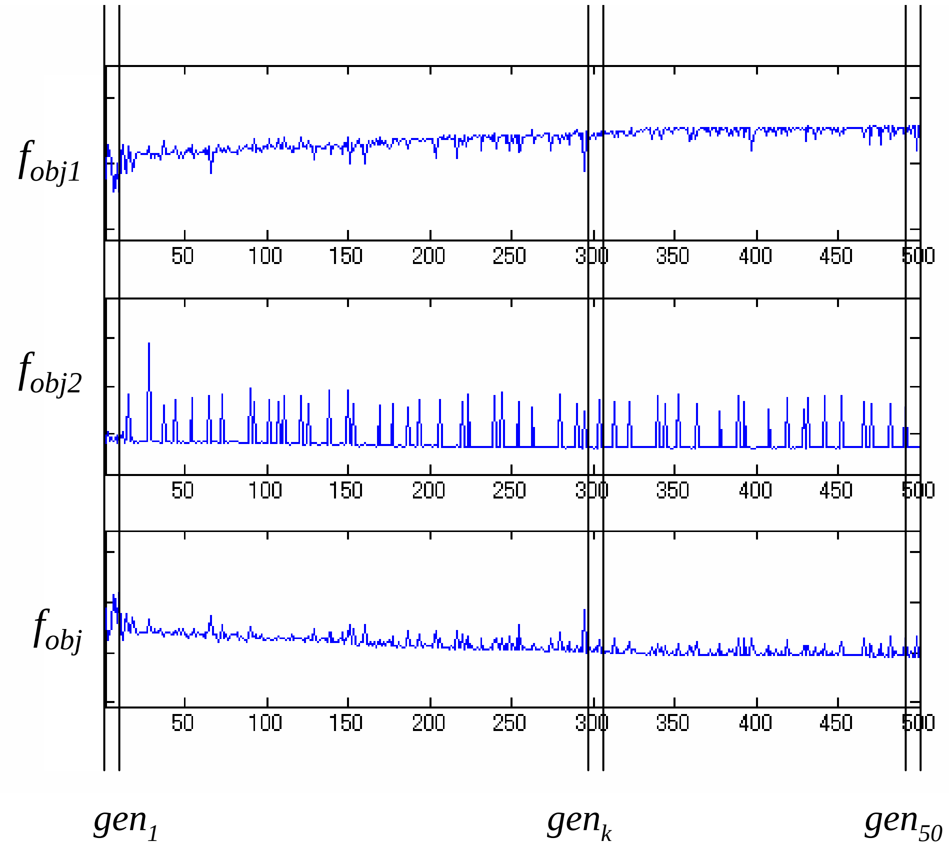
$w = 0.3$

$(1-w) = 0.7$

Review the objective function !!

$$f_{obj} = w1 \bullet \frac{1}{\sum_i \frac{N_i'}{N}} + w2 \bullet s \bullet \frac{E}{E_{\max}}$$

$$f_{obj} = w_1 \bullet \frac{1}{f_{obj1}} + w_2 \bullet (s \bullet f_{obj2})$$



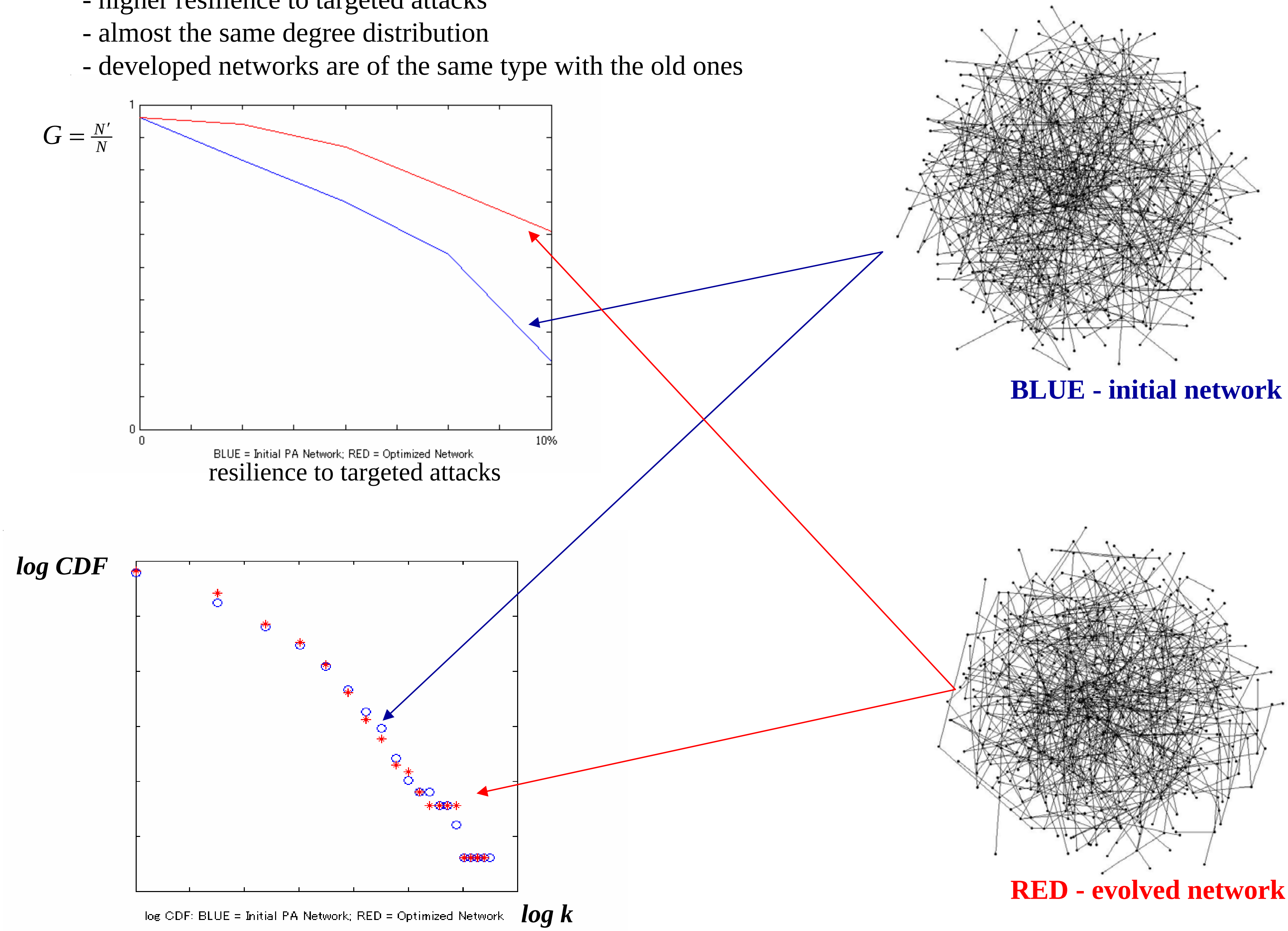
Performance of each individual:
- from 1st to 50th generation
- 500 individuals, in total

the population of networks evolves as follows:

- first objective:
  - increases visible
- second objective:
  - decreases very slow (almost constant)
- overall obj-function:
  - decreases visible

Should we expect for the network population to evolve into networks with the same structure, but better resilience ??

- **Sample network from the last generation has:**
  - higher resilience to targeted attacks
  - almost the same degree distribution
  - developed networks are of the same type with the old ones



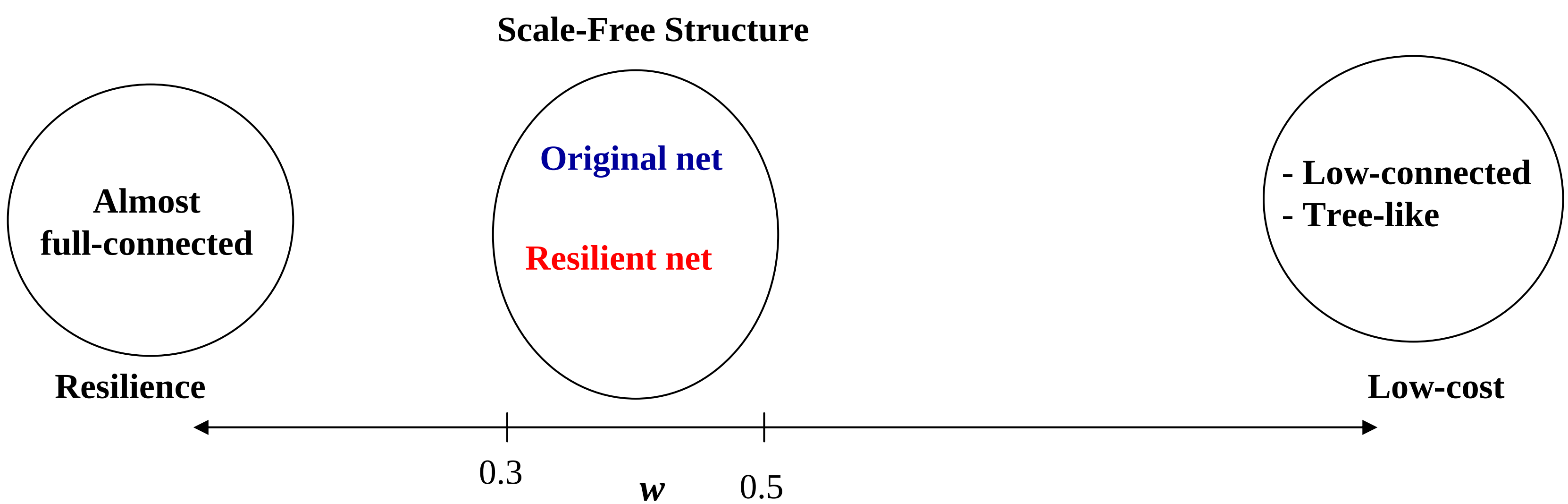
Conclusions

We demonstrated that a genetic-algorithm could be one viable solution for designing or optimizing large scale networks

- taking in account both robustness and cost, strong networks can be found, depending on one's interest for strength or cost.

The most important:

- for **specific values of *w*** and **appropriate number of generations** the original structure of the network can be preserved **in some limits**.



Future work

- different kinds of parameters & network attributes could be taken in account as "objective functions".
- also different kinds of network types could be subject of optimization, or could be analyzed from a multi-objective point of view.