The Use of Genetic Algorithms for Modelling the Behaviour of Boundedly Rational Agents in Economic Environments: Some Theoretical and Computational Considerations

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Complex Systems and Economic Modelling

Standard Models

- Individuals make decisions in isolation
- Only respond to information generated by the market
- Price system reflects all information (efficient marked hypothesis)

Complex System Models

- Interactions between individuals (among other things) determine the behaviour of the economic system.

- Individuals may interact in many different ways

Using a Genetic Algorithm (GA) to model the Behaviour of Boundedly Rational Agents.

- Individuals (artificially intelligent agents) revise their decisions according to a GA model of learning and adaption

- These 'artificially intelligent' agents are said to be 'boundedly rational'

Genetic Algorithm

- Random search techniques for finding good solutions
- Based on evolution of biological systems
- A population of N *chromosomes* (solutions) at time $t-1$ is selected to produce a 'mating pool' of size N by a process in which the probability of selection is higher for chromosomes with a higher *fitness* value.

Genetic Algorithm

- The genetic operators of *crossover* and *mutation* are applied to the mating pool to produce the new population of N chromosomes at time t. These operators are applied with a certain probability.

- A non-standard *elitist* operation, which we will use in the way suggested by Arifovic is sometimes also applied after selection, crossover and mutation. We will call this operator, *election*.

Genetic Algorithms - Model of Social Learning

- Riechmann (1999) proposed the following interpretation of the genetic operations in an economic context:

- Selection/Reproduction
  Learning by imitation

- Crossover
  Learning by communication

- Mutation
  Experimentation or perhaps making mistakes

T. Riechmann. Learning and behavioural stability: An economic interpretation of genetic algorithms. *Journal of Evolutionary Economics*, 9:225-242,1999.

Genetic Algorithms - Coding

- Suppose that the decision space of an economic agent is $\mathcal{D} = [0, M]$.

- To implement a GA we discretise $\mathcal{D}$.

- We denote the set of decisions (coded values) that result from this discretisation Ω.

- Coding mechanisms give rise to equally spaced possible decisions. We define the *grid* spacing Δ of a coding to be the difference between two consecutive decisions.

Binary Coding

For a binary code of fixed length l there will be $2^l - 1$ binary numbers with values

$$v_s = \sum_{j=1}^{l-1} 2^j a_j$$

where $a_j \in \{0, 1\}$, for all $i = 0, \ldots, l-1$.

These values are normalised so that $[0, M]$ is discretised into $2^l - 1$ coded values. Also

$$\Delta = \frac{M}{2^l - 1}.$$

Example: If the bit length of the binary string is ten, then 1,024 distinct grid points are spread equally over the domain $[0, M]$.

Genetic Algorithm - Fitness Function

- A coded decision is a member of the set Ω of coded decisions.

- A *population decision vector* is an ordered collection of N coded decisions $\Psi = \{\Psi_i\}$, $\Psi_i \in \Omega, i = 1, \ldots, N$.

- S is the space of all possible population decision vectors Ω^N.

Genetic Algorithm - Fitness Function

- For each $q \in \mathcal{D}$, we associate a real number which represents the fitness or payoff that an agent would receive conditional on some population decision vector $\Psi \in S$.

- Let $f_q(\Psi)$ be the continuous *fitness function* returning the fitness for any $q \in \mathcal{D}$.

Fitness Functions, GAs and Economic Models

- Dawid
  Mathematical analysis of GAs with a state dependent fitness function first given by Dawid.

H. Dawid. A Markov chain analysis of genetic algorithms with a state dependent fitness function. *Complex Systems* 8: 407 - 417, 1999.

GA Simulations of the Cobweb Model
Arifovic, Wheeler

Economy consists of N agents producing a single consumer good. The cost structure for the agents is identical.

$$c(q_i, t) = xq_{i,t} + \frac{1}{2}yNq_{i,t}^2$$

The price at time t is set by assuming a linear demand curve

$$p_t = A - B\sum_{i=1}^{N} q_{i,t}$$

where $A > 0$ and $B > 0$. For price to be positive, it is sufficient that

$$q_{i,t} < \frac{A}{B} \quad \text{for all} \quad i = 1, \ldots, N$$

We assume all agents have identical expectations p_t^e for the price p_t. Hence the expected profit is

$$\Pi_{i,t}^e = p_i^e q_{i,t} - x q_{i,t} - \frac{1}{2} y N q_{i,t}^2$$

Given $p_t^e > x$, agent i maximises $\Pi_{i,t}^e$ by choosing production

$$q_{i,t} = \frac{1}{yN}(p_t^e - x).$$

Rational expectations equilibrium

$$q^* = \frac{A - x}{N(B + y)}$$
$$p^* = \frac{Ay + Bx}{B + y}$$

Naive expectations: $p_t^e = p_{t-1}$

A first order constant coefficient linear difference equation is obtained for which the solution converges provided $B < y$.

GA Implementation of the Cobweb Model

Results from a Markov Chain Analysis

Theorem
Let q^* be the stationary equilibrium supply for the cobweb model. Consider a GA simulation with gridspacing Δ. If there is a supply decision $q \in \Omega$ such that

$$|q^* - q| < \frac{\Delta}{2} \frac{y}{B + y}$$

then the genetic algorithm will converge to the uniform state u_q with probability one. Conversely, if the genetic algorithm is convergent to a uniform state $u_q, q \in \Omega$ with probability one, then there must be a supply decision q such that the inequality is satisfied.

Cobweb model, GA simulation results (binary coding)

Number of agents	30
String length	10
Number of generations	10,000

Cobweb model, parameter sets 1 - 3

	A	B	x	y	N	B/y	q^*
Set 1	2.184	0.0152	0	0.016	30	0.95	2.3333
Set 2	2.184	0.015	0	0.0150206	30	0.9986	2.4250
Set 3	2.296	0.0168	0	0.0016	30	10.5	4.1594

Cobweb model, population decision vector, parameter set 1

	Vector
Set 1	$u_{(2.3339)}$

Cobweb model, population decision vector, parameter set 2.

Agents' decisions					
2.4226	2.4273	2.4226	2.4273	2.4273	2.4273
2.4273	2.4226	2.4273	2.4226	2.4226	2.4273
2.4226	2.4226	2.4273	2.4273	2.4226	2.4226
2.4226	2.4273	2.4226	2.4226	2.4273	2.4273
2.4273	2.4226	2.4273	2.4226	2.4273	2.4226

Work in Progress and Concluding Remarks

- We considered a GA (with an election operator) applied to a certain class of economic models

- A GA simulation provides a dynamical model for the evolution over time of the decisions of N interacting artificially intelligent agents

- We regard the successive set of decisions of the N agents given by the GA as arising from a discrete time Markov chain

Work in Progress and Concluding Remarks

- The properties of the transition matrix for this Markov chain depend on the economic model, the genetic operators and other parameters

- We establish a correspondence between the absorbing states of the Markov chain and economic equilibria

- As the GA of the Cobweb model illustrates, gridspacing (the distance between consecutive coded values) matters

Work in Progress and Concluding Remarks

- These considerations provided the basis for the paper
  A Markov analysis of social learning and adaptation (Wheeler, Bean, Gaffney and Taylor) J. Evol. Econ **16**: 299-319 (2006)

- Current work includes using Markov methodology to provide estimates of probabilities and tmies of various key transitions - such as transitions between neighbourhoods of different local maxima.